Topological Data Analysis

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University of Chicago

2016 REU

Challenges of Data

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- Size (“Big Data”)

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- Complexity

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  - Format

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- Size (“Big Data”)
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  - Format
  - Structure

Challenges of Complexity

Need organizing principles

Data has a shape

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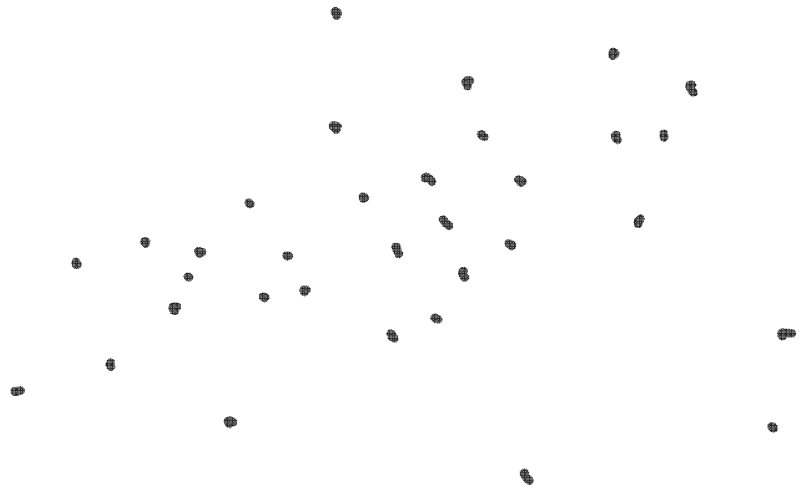

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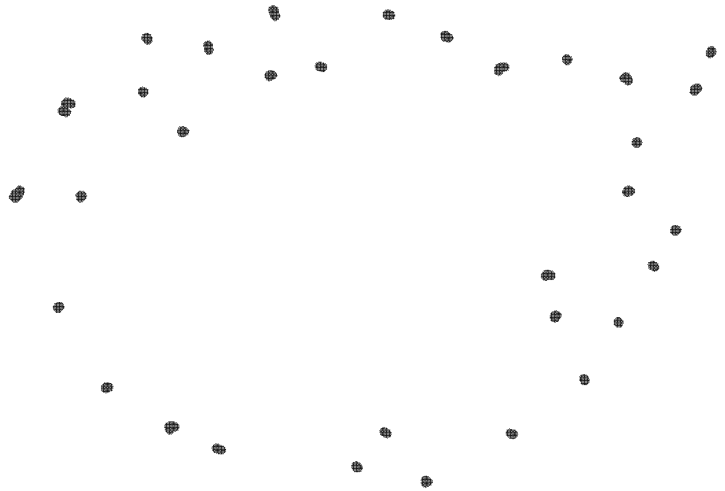

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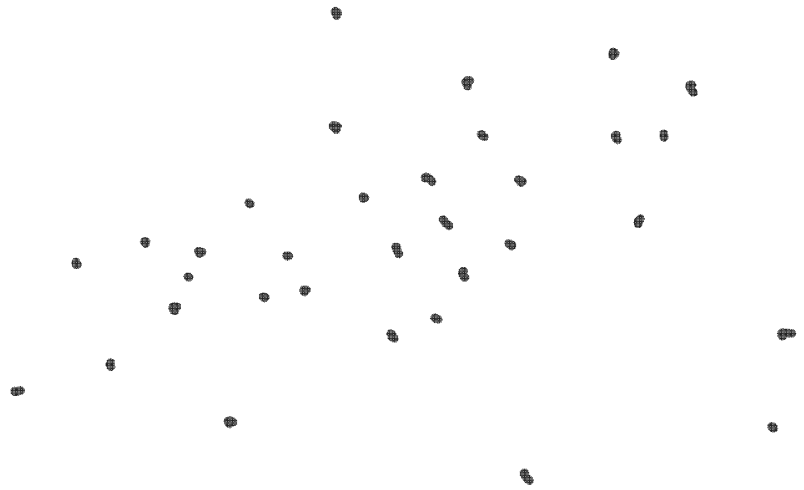

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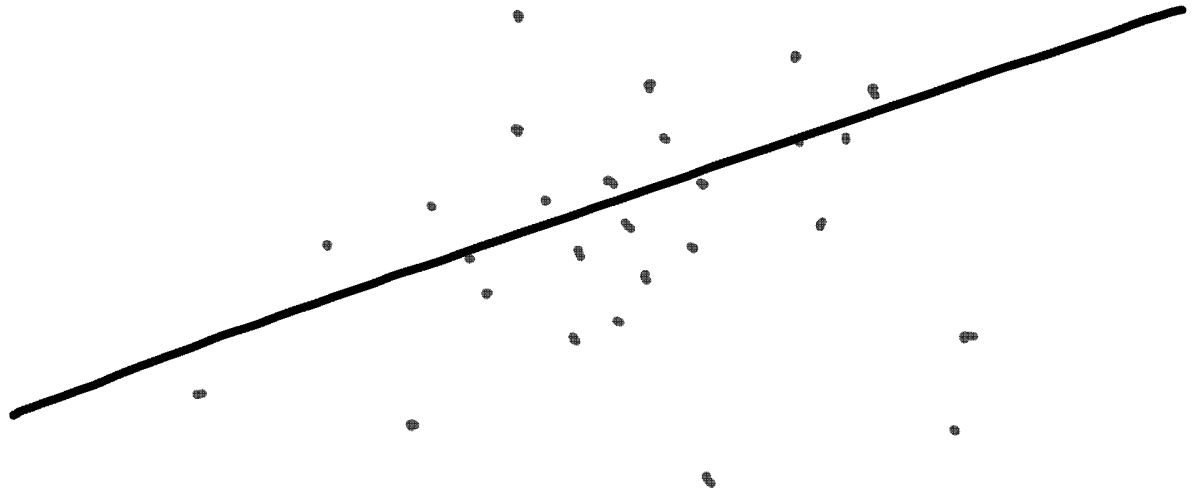
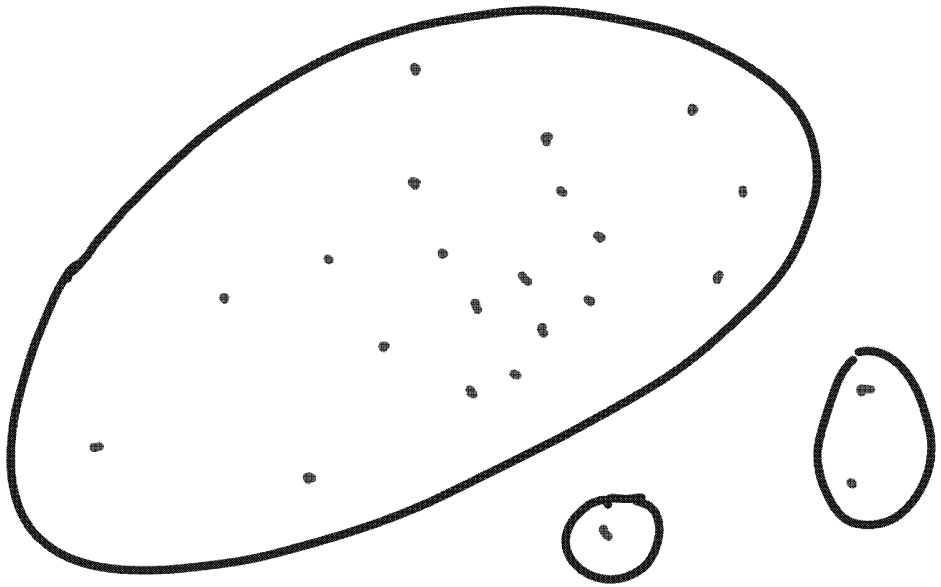


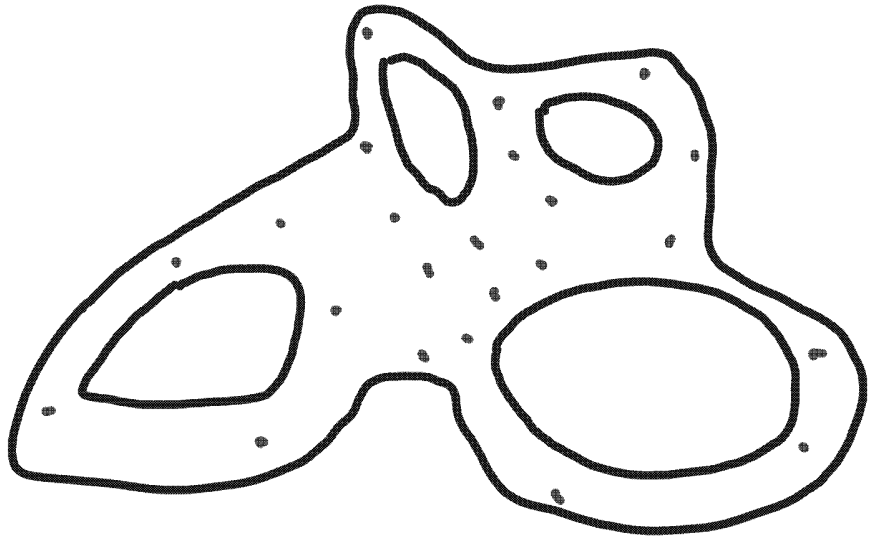
Linear

Data has shapes



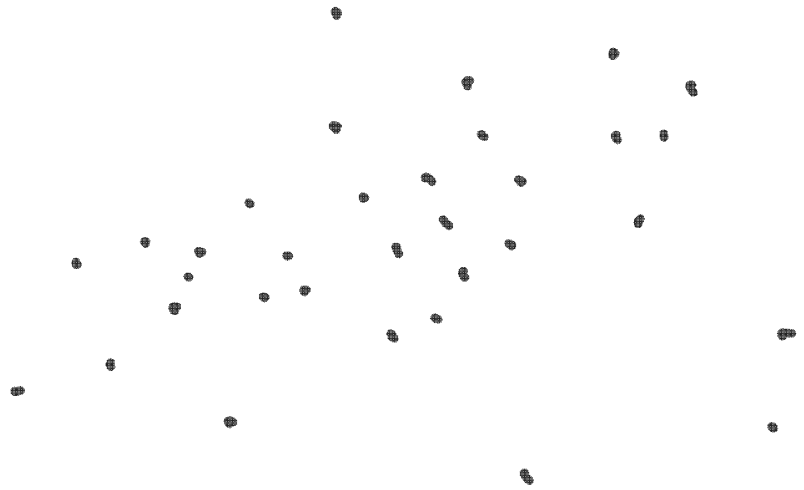
Clusters

Data has shapes



Holes

Data has shapes



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  - # of clusters

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  - # of holes
  - # of higher dimensional analogues

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X = Y

Connectivity – Advantages

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- Coordinate independent

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- Stable under small changes in the data

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- Coordinate independent
- Stable under small changes in the data
- Compressed representation

Persistent homology

- Homology

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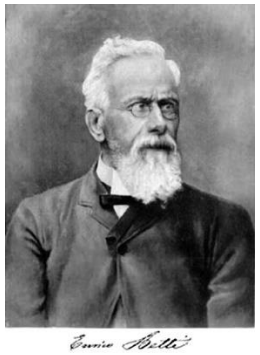
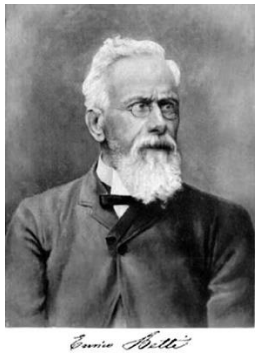


Enrico Betti

Persistent homology

- Homology



Enrico Betti

Emmy Noether

Photos from Wikipedia

Persistent homology

- Homology
  - Fundamental tool in algebraic topology

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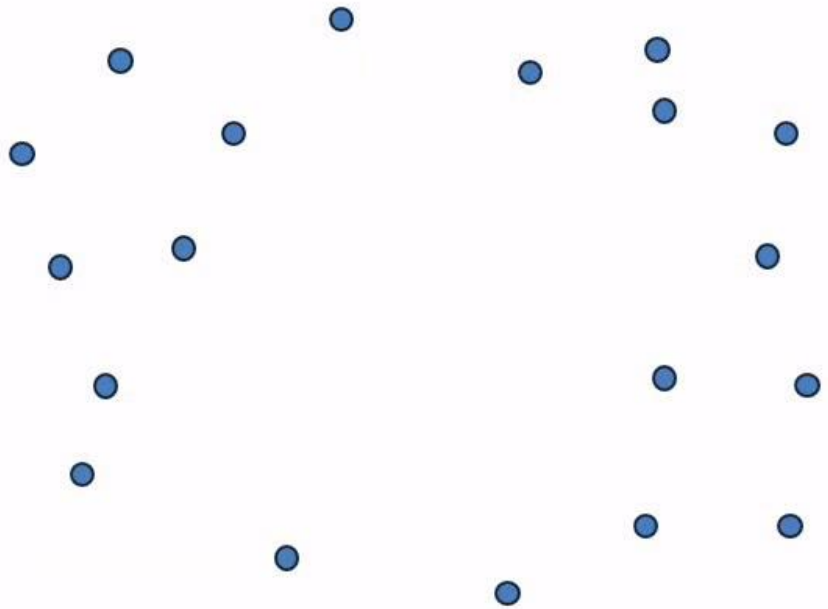
- Persistent
  - A data set gives rise to a sequence of spaces
  - Track where in the sequence clusters, holes, etc. appear, persist, disappear

Example:



Video by Matthew Wright, St. Olaf Collage

Outline

- Today
- Lecture 2
- Lecture 3

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  - Barcodes and persistent homology